



POWER LIMIT OF AN ENDOREVERSIBLE ERICSSON CYCLE WITH REGENERATION

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Abstract—An optimal power analysis is conducted on an endoreversible Ericsson cycle with perfect regeneration. The endoreversible cycle is one in which the external heat transfer processes are the only irreversible processes of the cycle. Maximum power and efficiency at maximum power are obtained for the cycle, based upon the higher and lower temperature bounds. These results provide additional criteria for use in the study and performance evaluation of Ericsson engines.

Heat engine Ericsson cycle Endoreversible Power

NOMENCLATURE

- A_H = Heat transfer surface area between endoreversible Ericsson engine and high temperature source
- A_L = Heat transfer surface area between endoreversible Ericsson engine and low temperature sink
- C = Heat engine parameter defined by equation (22)
- C_p = Specific heat for constant pressure process
- m = Mass of working fluid in engine
- P = Net cycle power output
- $P_{\max}$ = Maximum cycle power output
- p_1 = Pressure at bottom dead center
- p_2 = Pressure at top dead center
- Q_{in} = Isothermal heat transfer into engine from high temperature source, per cycle
- Q_{out} = Isothermal heat transfer from engine to low temperature sink, per cycle
- ${}_1Q_2$ = Isothermal heat transfer from engine to low temperature sink, per cycle
- ${}_2Q_3$ = Heat transfer from regenerator to working fluid
- ${}_3Q_4$ = Isothermal heat transfer from engine to low temperature sink per cycle
- ${}_4Q_1$ = Heat transfer captured by regenerator from working fluid
- $\dot{Q}_{in}$ = Rate of heat transfer into engine from high temperature source
- $\dot{Q}_{out}$ = Rate of heat transfer from engine to low temperature sink
- R = Gas constant
- r_p = Pressure ratio, p_2/p_1
- s = Specific entropy
- T_c = Temperature of working fluid during isothermal external heat rejection process to low temperature sink
- T_H = Temperature of heat source
- T_L = Temperature of heat sink
- T_w = Temperature of working fluid during isothermal external heat input process from high temperature source
- t = Total cycle time
- t_{12} = Time required to accomplish isothermal compression
- t_{23} = Time required for heat transfer process from regenerator to working fluid
- t_{34} = Time required to accomplish isothermal power stroke
- t_{41} = Time required for heat transfer process from working fluid to regenerator
- U_H = Overall heat transfer coefficient between engine and high temperature source
- U_L = Overall heat transfer coefficient between engine and low temperature sink
- W_{net} = Net work output for one cycle
- W_{opt} = Net work output per cycle at optimum power condition
- η_{opt} = Cycle efficiency at optimum power condition

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INTRODUCTION

Because of its potential for providing a thermal energy efficiency which can, in theory, be made to approach the Carnot efficiency, the Ericsson engine cycle is one of a handful of regenerative thermodynamic cycles that have captivated several generations of engineers and physicists during the past two centuries. It is named after the Swedish inventor who first introduced the world to an engine with its features in 1833. Subsequently, many additional inventors became engaged in producing a variety of engine innovations based on the thermodynamic principles of this cycle. Most of these devices were small, but some were large also. Most notable perhaps of the latter type was a marine engine built in 1853 by Ericsson himself. This engine had four 11 ft dia cylinders and a 5 ft stroke. It ran at 9 rpm and produced 300 b.h.p. However, the advent of other innovations, coupled with the lack of appreciation for the potential of regeneration on the part of most thermodynamicists of the period, caused a wane in interest in this engine during the latter part of the nineteenth century, which carried over into the present century. Fortunately, interest in regenerative type external heat engines was revived in the late 1930s by the Phillips company. Since then, the many strong advantages of the Ericsson cycle, such as its low noise and air pollution levels, its realizable potential for high thermal efficiencies and its flexibility as an external heat engine to utilize a variety of energy sources, have all worked together to propel it back into the attention of the world [1, 2].

Today, the world community is looking for fuel efficient and environmentally viable alternatives for many of the traditional energy conversion approaches. This development has further worked to increase the technical focus on the Ericsson cycle, and presently, the momentum of its popularity is continuing to accelerate. In recent years, the use of the Ericsson cycle has been considered in electrothermodynamic power convertors [3], solar-powered heat engines [4], refrigeration applications [5–8], MHD thermal-electric generators [9, 10], waste heat recovery systems [11], infra-red electronics cooling [12], airconditioning and heating [13–15], etc. The cycle, in its reversed mode, is being found to be one of the viable approaches for the replacement of environmentally unacceptable halogen based refrigerants [7, 14]. Exploration of its potential use has ranged from extra-terrestrial [10–13] to subterranean [16] applications.

A distinction between Ericsson and Stirling cycle machines has not been well presented in the literature of both, with the meaning of the latter often being understood to include a closed cycle version of the former. However, in thermodynamic terms, the Ericsson cycle is considered to be either an open or closed cycle in which the primary heat addition and rejection processes can be modeled as taking place at constant temperatures, and the cycle expansion and compression processes can be modeled as occurring at constant pressures. Currently, practical engineers make use of the ideal air standard Ericsson cycle as a basis for analyzing trends within real versions of Ericsson engines. In general, however, this type of thermodynamic analysis provides exceedingly generous estimates of the potential performance of these heat engines [7]. Thus, in order to provide a more reasonable estimate of the performance potential of a real Ericsson engine, an endoreversible Ericsson engine cycle is studied herein, using a finite time thermodynamic analysis [18]. This innovative approach is based upon the inherent reality that a heat engine must produce work in a limited amount of time. In order to accomplish this task, the external heat transfer processes of the cycle must occur across finite temperature differences and must, thereby, be irreversible. The endoreversible cycle is, therefore, optimized with respect to power, not efficiency, as has been the usual focus of thermodynamic analysis. For clarity, this study is, thus, based upon the recognition that real engines must produce power and not simply work over an infinite time. The results of the analysis contained in this paper are intended for use in providing insight into the design and operating criteria necessary to attain optimum power in Ericsson engines with regeneration.

ENDOREVERSIBLE ERICSSON CYCLE WITH REGENERATION AND ITS POWER OPTIMIZATION

The regenerative endoreversible Ericsson cycle is depicted in Figs 1 and 2. This cycle approximates the compression stroke of the real engine as an isothermal process, 1–2, with irreversible heat rejection to a low temperature sink. The heat addition to the working fluid from the regenerator is modeled as a reversible constant pressure process, 2–3. The work producing

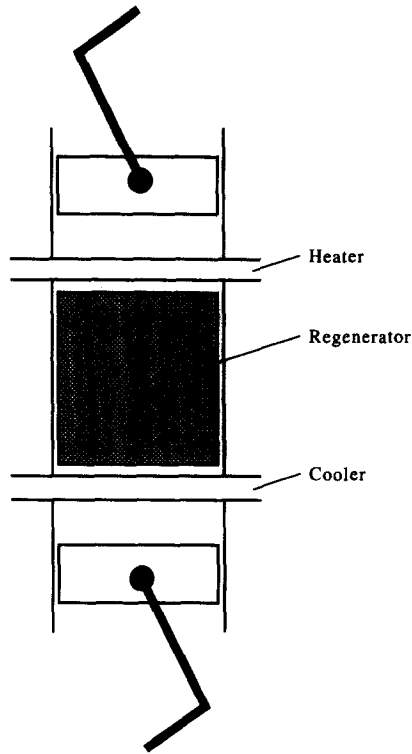


Fig. 1. Functional schematic diagram of regenerative Ericsson engine (note that the opposing pistons are 90° out of phase).

expansion stroke is modeled as an isothermal process, 3–4, with irreversible heat addition from a high temperature source. Finally, the heat rejection to the regenerator is modeled using a reversible constant pressure process, 4–1.

As alluded to earlier, the external heat transfer processes, 1–2 and 3–4, within real Ericsson engines must each occur in finite time. This, in turn, requires that these heat transfer processes must proceed through finite temperature differences and are, therefore, defined as being externally irreversible. During the first of these, the heat rejection process 1–2, energy flows from the working fluid which is maintained at a constant temperature, T_c , to the low temperature sink, T_L ; thus flowing across the temperature difference $(T_c - T_L)$, as shown in Fig. 2. Similarly, during the heat addition process, 3–4, heat is transferred from the high temperature source, T_H , to the working

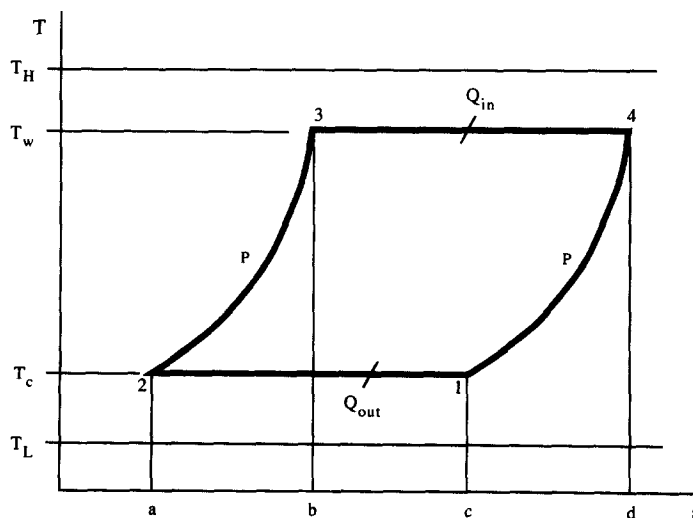


Fig. 2. Temperature-entropy diagram of the regenerative Ericsson engine (ideal).

fluid which is maintained at a constant temperature, T_w ; thus flowing across the temperature difference ($T_H - T_w$).

If the regeneration is assumed to be ideal, the heat rejected to the regenerator by the hot working fluid during the process 4-1 is equivalent to the heat supplied to the cold working fluid during 2-3. This can be visualized by noting that the area under the process path on the temperature-entropy diagram of Fig. 2 for process 4-1, area c-1-4-d-c, is equal in magnitude to the area under process path 2-3, area a-2-3-b-a. This assumption is reasonable, since the efficiency of regenerators is continuing to improve, with at least one manufacturer as far back as 1972 reporting regenerator effectiveness values of 95% for Stirling type engines [19], and progressing to current values of 98-99% [20]. Thus, the following relation is written for the overall ideal regenerative heat transfer processes,

$$|{}_2Q_3| = |{}_4Q_1|. \quad (1)$$

Since the regenerator supplies the heat transfer to the working fluid during 2-3, the only source of external heat addition is provided by the source from 3-4. If the working fluid is considered to be an ideal gas, the magnitude of the heat addition during 3-4 can be found by substituting the definition for boundary work into the conservation of energy relation. This yields

$$Q_{in} = {}_3Q_4 = mRT_w \ln r_p \quad (2)$$

where r_p is the pressure ratio, p_2/p_1 .

Similarly, the external heat rejection, 1-2, is found to be

$$Q_{out} = -{}_1Q_2 = mRT_c \ln r_p. \quad (3)$$

For the thermodynamic cycle, 1-2-3-4-1, conservation of energy yields

$$W_{net} = Q_{net} = Q_{in} - Q_{out}. \quad (4)$$

Substituting equations (2) and (3) into equation (4) gives

$$W_{net} = mR \ln r_p (T_w - T_c). \quad (5)$$

The average net power for one cycle can be found by dividing the net cycle work by the time for one cycle, t . This approach yields the following equation for the net cycle power,

$$P = \frac{W_{net}}{t} = \frac{mR \ln r_p (T_w - T_c)}{t}. \quad (6)$$

The total cycle time is the sum of the individual process times,

$$t = t_{12} + t_{23} + t_{34} + t_{41}. \quad (7)$$

Using equations (2) and (3), the average rates of external heat transfer into and out of the engine can be quantified using thermodynamic theory as,

$$\dot{Q}_{in} = \frac{Q_{in}}{t_{34}} = \frac{mRT_w \ln r_p}{t_{34}} \quad (8)$$

and

$$\dot{Q}_{out} = \frac{Q_{out}}{t_{12}} = \frac{mRT_c \ln r_p}{t_{12}}. \quad (9)$$

The rates of external heat transfer into and out of the engine may further be quantified from heat transfer theory as being proportional to the temperature difference between the respective thermal reservoirs and the constant temperature states of the working fluid during each process. This implies that equations (8) and (9) can also be expressed as:

$$\dot{Q}_{in} = U_H A_H (T_H - T_w) \quad (10)$$

and

$$\dot{Q}_{out} = U_L A_L (T_c - T_L) \quad (11)$$

where each U represents the overall heat transfer coefficient and A is the associated heat transfer area.

Substituting equations (10) and (11) into equations (8) and (9), respectively, yields expressions for the times associated with the external heat transfer processes,

$$t_{12} = \frac{mRT_c \ln r_p}{U_L A_L (T_c - T_L)} \quad (12)$$

and

$$t_{34} = \frac{mRT_w \ln r_p}{U_H A_H (T_H - T_w)}. \quad (13)$$

The times associated with the regenerative heat transfer processes can be found by modeling the regenerator as a cross flow heat exchanger. For such

$$t_{23} = t_{41} = \frac{mC_p (T_w - T_c)}{U_{\text{reg}} A_{\text{reg}} (\text{LMTD}_{\text{reg}})} \quad (14)$$

where LMTD_{reg} is the log-mean temperature difference,

$$\text{LMTD}_{\text{reg}} = \frac{(T_w - T_{\text{cl}}) - (T_{\text{wl}} - T_c)}{\ln \left(\frac{T_w - T_{\text{cl}}}{T_{\text{wl}} - T_c} \right)}$$

where

$$\begin{aligned} T_c &= \text{Cold temperature working fluid inlet} \\ T_{\text{cl}} &= \text{Cold temperature working fluid outlet} \\ T_w &= \text{Warm temperature working fluid inlet} \\ T_{\text{wl}} &= \text{Warm temperature working fluid outlet.} \end{aligned}$$

A regenerator effectiveness of 100% implies that the outlet temperatures of the heat exchanger will equal the inlet temperature, ($T_{\text{cl}} = T_w$, $T_{\text{wl}} = T_c$). To determine the effect that this has on the LMTD of the regenerator, a numerical analysis was conducted in which the outlet temperatures of the regenerator were allowed to approach the inlet temperatures, ($T_{\text{cl}} \rightarrow T_w$, $T_{\text{wl}} \rightarrow T_c$). This analysis concluded that, at the limit of 100% effectiveness, the LMTD of the regenerator becomes unity,

$$\text{LMTD}_{\text{reg}} = 1 \text{ K}. \quad (15)$$

Finally, the expression for the process times associated with the regenerative heat transfer is,

$$t_{23} = t_{41} = \frac{mC_p (T_w - T_c)}{U_{\text{reg}} A_{\text{reg}} (1 \text{ K})}. \quad (16)$$

Equations (13) and (14) can then, in turn, be substituted into equation (6), yielding an expression for the power which is based upon the respective reservoir and the isothermal working fluid temperatures,

$$P = \left[\frac{T_w}{U_H A_H (T_H - T_w) (T_w - T_c)} + \frac{T_c}{U_L A_L (T_c - T_L) (T_w - T_c)} + \frac{2C_p}{R \ln r_p U_{\text{reg}} A_{\text{reg}}} \right]^{-1}. \quad (17)$$

The maximum power with respect to the two, as yet undetermined working fluid temperatures, T_w and T_c , can be obtained through simultaneous solution of the following equations:

$$\left(\frac{\partial P}{\partial T_w} \right) T_c = 0 \quad (18)$$

and

$$\left(\frac{\partial P}{\partial T_c} \right) T_w = 0. \quad (19)$$

This approach yields the warm and cold constant temperature states of the working fluid which will provide maximum power. These temperatures are found as functions of the temperatures of the high and low thermal reservoirs as shown below:

$$(T_c)_{\text{opt}} = CT_L^{0.5} \quad (20)$$

and

$$(T_w)_{\text{opt}} = CT_H^{0.5} \quad (21)$$

where

$$C = \frac{(U_H A_H T_H)^{0.5} + (U_L A_L T_L)^{0.5}}{(U_H A_H)^{0.5} + (U_L A_L)^{0.5}}. \quad (22)$$

These values were verified as yielding the maximum power by ensuring that the sign of the second derivative of power with respect to each of the fluid temperatures, warm and cold, were negative after substituting the optimum values.

The expression for maximum power is, thus:

$$P_{\text{max}} = \left[\frac{[(U_L A_L)^{0.5} + (U_H A_H)^{0.5}]^2}{(U_L A_L)(U_H A_H)(T_H^{0.5} - T_L^{0.5})^2} + \frac{2C_p}{(U_{\text{reg}} A_{\text{reg}})R \ln r_p} \right]^{-1}. \quad (23)$$

The work per cycle at maximum power, or work optimum, is determined by substituting the optimum working fluid temperature values, equation (20) and (21), into the work equation (5). This results in the following equation:

$$W_{\text{opt}} = CmR \ln r_p [T_H^{0.5} - T_L^{0.5}] \quad (24)$$

where C is defined, as before, by equation (22).

Finally, the thermal efficiency of the cycle at the location of maximum power is found by first substituting the optimum warm fluid temperature, equation (21), into equation (2) for the external heat transfer to the working fluid. This result and the optimum work, equation (24), are then substituted into the basic definition for thermal efficiency yielding:

$$\eta_{\text{opt}} = \frac{W_{\text{opt}}}{(Q_{\text{in}})_{\text{opt}}} = \frac{R \ln r_p [(T_H)^{0.5} - (T_L)^{0.5}]}{(T_H)^{0.5} R \ln r_p}. \quad (25)$$

This result can then be simplified to

$$\eta_{\text{opt}} = 1 - \left[\frac{T_L}{T_H} \right]^{0.5}. \quad (26)$$

SAMPLE OPTIMUM POWER CALCULATIONS

A parametric study was conducted to observe the effects of different source temperatures and overall heat transfer parameters, $U_L A_L$ and $U_H A_H$, on the maximum power defined by equation (23). The intent of this study is to demonstrate trends; however, the authors have tried to provide reasonable values for use in the expressions. Although many Ericsson and Stirling type engines use different working fluids, air was used in this study to maintain compatibility with existing air standard cycle calculations. The value for C_p used in the calculations was 1.003 kJ/kgK, and the pressure ratio was set to 2.639. The sink temperature was chosen to be 300 K to match typical ambient conditions. Using representative data for Stirling type engines from West [21], the average heat transfer parameter for the regenerator, $U_{\text{reg}} A_{\text{reg}}$, was set at 1000 kW/K, while the coefficients for the heater and the cooler overall heat transfer parameters ranged from 1–4 kW/K. The working fluid mass used in the calculations was 9.3×10^{-3} kg and the R value for air is 0.287 kJ/kgK. To maintain time symmetry between the external heating process, 3–4, and the cooling process, 1–2, the values for $U_L A_L$ were set equal to those used for $U_H A_H$ and were then varied together over the range 1–4 kW/K. For each heat transfer parameter value, the source temperature was varied from 900 to 1500 K to encompass the range of operating temperatures of real engines. The results of this study are presented in Fig. 3.

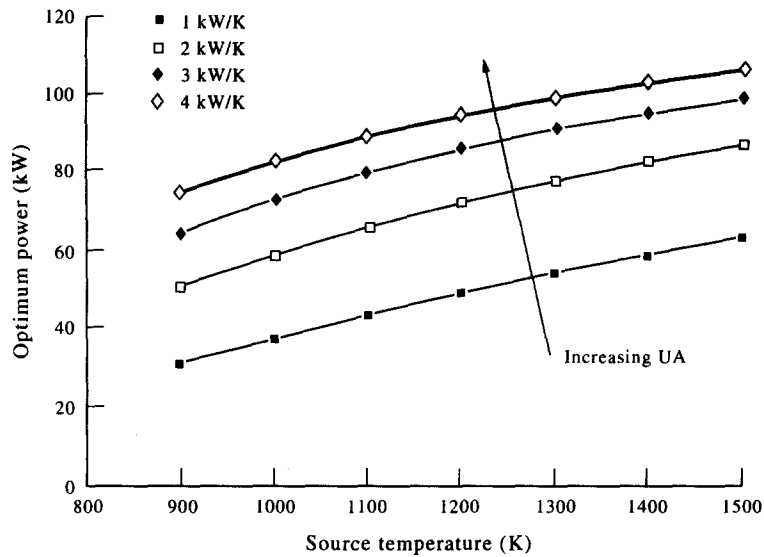


Fig. 3. Optimum power vs source temperature for various $U_H A_H$.

Figure 3 demonstrates that increasing the source temperature will provide an increase in the optimum power output of the cycle. This should be expected, since the difference between the source and sink temperatures will be increasing, thus providing a higher potential for enhanced heat transfer rates through the engine. Likewise, raising the overall heat transfer parameters, $U_L A_L$ and $U_H A_H$, also provides greater heat transfer rates for a given set of operating temperatures. These higher heat transfer rates are then translated into higher optimum power outputs by the engine.

Further insight into the design of regenerative Ericsson engines can be obtained using equations (20)–(22). These equations relate the optimum design working fluid temperatures in the engine to the source and sink temperatures and to the overall heat transfer parameters of the heat exchangers. Using these equations during the design process, in concert with the maximum power relation [equation (23)], can help in the selection of heat exchangers and working fluid conditions.

CONCLUSIONS

The design of regenerative Ericsson engines has challenged many engineers. Unfortunately, ideal Ericsson cycle analysis using thermodynamic techniques alone yields overly generous estimates of the potential performance of these engines. The ideal Ericsson cycle analysis is too demanding, in that it requires all processes to be completely reversible. This leads to infinitely large cycle times and, thus, to the production of zero power. Stated differently, a traditional Ericsson cycle thermodynamic analysis provides relations for optimum efficiency and work per cycle at optimum efficiency. However, the cycle will require an infinite time for completion, providing zero power output at optimum efficiency. Obviously, this is unacceptable for real engines. This paper analyzes an endoreversible Ericsson cycle using finite time thermodynamics to provide a more reasonable estimate of the potential performance bounds for real engines. The relations thus derived are optimized to provide maximum power. The results of this analysis provide insight into the design of Ericsson engines to achieve maximum power output.

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